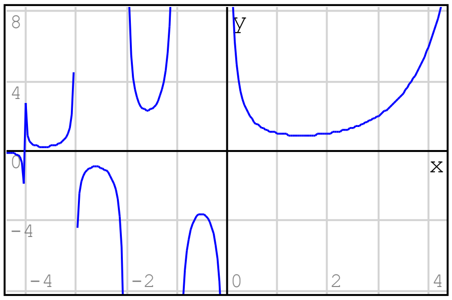
H. T. Davis Γa(x)

Lanczos Gamma Approximations



$\Gamma(x)$

$\varepsilon := 10^{-17}$

$\left[\begin{array}{c} \text{appVersion}(4) = \text{"0.99.7822.147"} \\ Applications = \text{" 7 s "} \end{array}\right]$

Sanity check Mathcad 11

https://keisan.casio.com/exec/system/1180573447

$$\begin{bmatrix} \Gamma(1;\ \varepsilon) \\ \Gamma(a;\ \varepsilon) \end{bmatrix} = \begin{bmatrix} 1.000000000000000 \\ 1.772453844580960 \end{bmatrix}$$

● γ(a,x)	1.2100356193111
● Γ(a,x)	0.56241823159441

$$\begin{bmatrix} \gamma(0.5;\ 0.5) \\ \Gamma(0.5;\ 0.5) \\ \Gamma(14.5) \end{bmatrix} = \begin{bmatrix} 1.21003561931112 \\ 0.5624182315944 \\ 2.30923179223141 \cdot 10^{10} \end{bmatrix}$$

NOTE ... Smath plots at the kernel level
but solves neither γ(a,x), Γ(a,x).
It complains about 'k'...
f(x) reconciliates the solver.

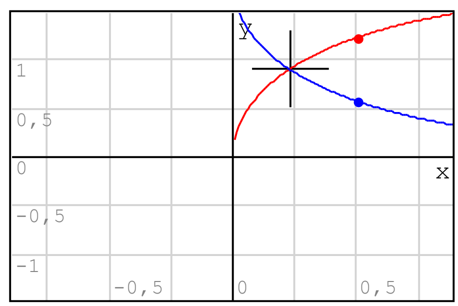
$a := 0.5$

$$\lambda(x) := \begin{cases} 1 & \text{if } (0 \le x) \wedge (x \le 2 \cdot a) \\ \text{""} & \text{otherwise} \end{cases}$$

$$Jnct := \frac{\Gamma(a;\ a) + \gamma(a;\ a)}{2} = 0.8862$$

$$\begin{array}{l} X := [0.1;\ 0.11..a] \\ \text{for } i \in [1..\text{rows}(X)] \\ \quad Y_i := \Gamma(a;\ X_i) \\ f(x) := \text{cinterp}(X;\ Y;\ x) \end{array}$$

$$U := \text{solve}\left(f(x) - Jnct = 0;\ x;\ X_1;\ a\right) = 0.227468207128138$$



Example_1: Multistage XFR

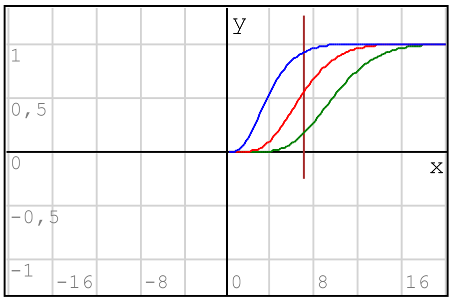
$t0 := \mathrm{time}(1)$

$T := 1$ System TimeConstant $\quad s := 1$ Laplace UnitStep

$$\lambda(x) := \begin{cases} 1 & \text{if } (0 \le x) \wedge (x \le 20) \\ "" & \text{otherwise} \end{cases} \qquad time := 7$$

$$f(n; t) := \frac{\Gamma(n+1) - \Gamma\left(n+1; \frac{t}{T}\right)}{s \cdot \Gamma(n+1)}$$

Laplace XFR multistage system 'n' stages, 'T' time constant



$$\begin{bmatrix} f(3; time) \\ f(6; time) \\ f(9; time) \end{bmatrix} = \begin{bmatrix} 0.92 \\ 0.55 \\ 0.17 \end{bmatrix} \qquad \begin{bmatrix} 0.92 \\ 0.55 \\ 0.17 \end{bmatrix}$$

Example_2: Fitting Mathsoft data set.

$$XY := \begin{bmatrix} 0 & 127.42 \\ 0.1 & 123.469 \\ 0.2 & 118.008 \\ 0.3 & 111.187 \\ 0.4 & 102.811 \\ 0.5 & 93.554 \\ 0.6 & 84.168 \\ 0.7 & 74.818 \\ 0.8 & 66.373 \\ 0.9 & 58.587 \\ 1 & 51.678 \\ 1.1 & 45.739 \\ 1.2 & 40.675 \\ 1.3 & 36.404 \\ 1.4 & 32.74 \\ 1.5 & 29.581 \\ 1.6 & 26.848 \\ 1.7 & 24.472 \\ 1.8 & 22.395 \\ 1.9 & 20.573 \\ 2 & 18.97 \\ 2.1 & 17.555 \\ 2.2 & 16.3 \\ 2.3 & 15.184 \\ 2.4 & 14.188 \\ 2.5 & 13.295 \\ 2.6 & 12.492 \\ 2.7 & 11.768 \\ 2.8 & 11.12 \\ 2.9 & 10.547 \\ 3 & 10.028 \end{bmatrix}$$

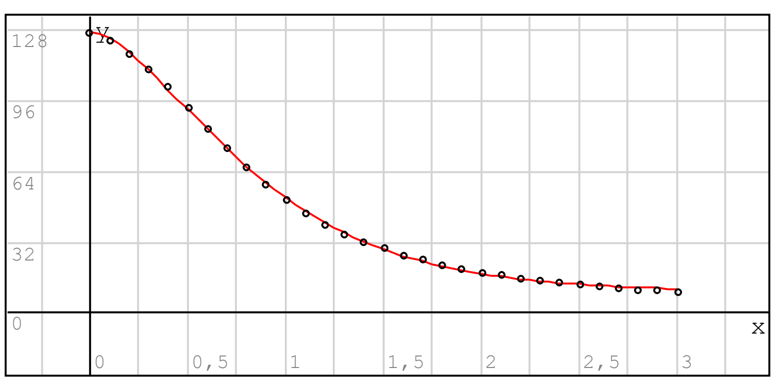
$$plot(data; char; size; clr) := \left| \begin{array}{l} \text{for } k \in [1..\mathrm{rows}(data)] \\ \quad \left[r3_k := char \quad r4_k := size \quad r5_k := clr \right] \\ \mathrm{augment}(data; r3; r4; r5) \end{array} \right.$$

$pts := plot(XY; "o"; 5; "black")$

$[\, yo := 10.028 \quad a := 143 \quad b := 1.77 \quad n := 2.5 \,]$ $\qquad \varepsilon := 10^{-15}$

$$f(x) := yo + a \cdot \Gamma(b; x)^n$$

$$\Omega := \left| \begin{array}{l} u := [\varepsilon; 0.05..3] \\ \text{for } i \in [1..\mathrm{rows}(u)] \\ \quad vy_i := \mathrm{eval}(f(u_i)) \\ \mathrm{augment}(u; vy) \end{array} \right.$$



$[\, yo := 10.028 \quad a := 143.017 \quad b := 1.7666 \quad n := 2.486 \,]$ $\qquad \mathrm{time}(1) - t0 = 7.2\ s$

$a := 0.5$

"RootSecant" is viable => guess wisely tial/error

$$X(\Phi; \alpha; \beta; \varepsilon) := \left| \begin{array}{l} \text{"Root Secant"} \\ u_1 := \varepsilon \\ u_2 := \beta - \dfrac{\Phi(\beta)\cdot(\alpha-\beta)}{\Phi(\alpha)-\Phi(\beta)} \\ i := 2 \\ u_{i+1} := u_i - \dfrac{\Phi(u_i)\cdot(u_{i-1}-u_i)}{\Phi(u_{i-1})-\Phi(u_i)} \\ \text{while } \left|u_i - u_{i-1}\right| \geq \varepsilon \\ \quad \left| \begin{array}{l} u_{i+1} := u_i - \dfrac{\Phi(u_i)\cdot(u_{i-1}-u_i)}{\Phi(u_{i-1})-\Phi(u_i)} \\ i := i+1 \end{array} \right. \\ u \end{array} \right.$$

$\Phi(x) := \mathrm{eval}(\gamma(a; x) - \Gamma(a; x))$

$[\alpha := 0.2 \;\; \beta := 0.5 \;\; \varepsilon := 10^{-15}]$

$sol := X(\Phi; \alpha; \beta; \varepsilon)$

$$sol = \begin{bmatrix} 1\cdot10^{-15} \\ 0.238739753660939 \\ 0.23385967247468 \\ 0.227353946214894 \\ 0.227469375974804 \\ 0.227468211772556 \\ 0.227468211559788 \\ 0.227468211559788 \end{bmatrix}$$

0.22746820712813